Announcements

Last time we described the trajectory of a typical star on the HR diagram, focusing on the pre-main-sequence phase and life on the MS. When a star exhausts the hydrogen in its core it begins the relatively short final phase of its life. It goes through a series of collapses after which the next nuclear furnace is ignited, depending on the star’s mass, all the while the outer layers are puffed up so that the star is in the red giant part of the HR diagram. It next passes through the variable stage, to be discussed briefly below. When all else fails the star collapses to its final resting place, depending on the final mass of the star: white dwarf, neutron star or black hole.

Regions of HR diagram

Typical evolution of 1 Mₖ star

White dwarf: as a small star collapses its matter gets closer and closer together until what stops the collapse is the dense sea of outer electrons; electrons repel each other and more than that cannot be packed too close together—their pressure prevents the star from collapsing further. White dwarfs are incredibly dense: a typical density is 1000 tons/tablespoon. Think of a white dwarf as the mass of the sun packed into the radius of earth. There is a maximum mass of about 1.4 Mₖ for a white dwarf; this is called the Chandrasekhar limit. An example of a white dwarf is Sirius B, first seen in 1862.

Neutron stars: for stellar final masses mₖ between 1.4 Mₖ and 2.5 Mₖ, that is 1.4 Mₖ < mₖ < 2.5 Mₖ, the electrons essentially combine with the protons to make neutrons and so the star can collapse below the white dwarf limit all the way down to a compact sea of neutrons. But neutrons also have a pressure so that the collapse is finite. A neutron star is typically 2 solar masses in a sphere of radius 20 km, with a density of 10 billion tons per tablespoon! As these stars collapse so far, they usually ‘spin up’ and have high rotation rates and magnetic fields; the rapid rotation is analogous to an ice skater spinning slowly with arms extended holding weights—when she quickly brings the weights in by bending her arms, her body spins up very rapidly (basically due to conservation of angular momentum). A special class of neutron star is the pulsar whose rapid rotation combined with inclination of magnetic field to rotation axis (oblique rotator) makes a lighthouse-like flashing or pulsating of its
radio or light output. Typical rotation periods for pulsars are 0.01 to 0.001 seconds.

Black Holes: Stars with final masses > 2.5 \( M_\odot \) will continue to collapse past the neutron star stage, all the way down until they are so incredibly dense light itself cannot leave the star. They are still detectable from their gravitational influence on other objects, which can orbit them, or slowly spiral in emitting light or other radiation, such as x-ray emission. We can calculate the radius of a Black Hole as a function of its mass by setting the kinetic energy of a photon (\( \frac{1}{2} m c^2 \)) equal to the potential energy at the edge of the black hole (\( GMm/r_\bullet \)), where \( r_\bullet \) is called the Schwarzschild radius. The answer is \( r_\bullet = \frac{2GM}{c^2} \), the radius within which an object must shrink to become a black hole. Putting in the constants, \( r_\bullet = 6 \text{ km} \ (M/M_\odot) \).

Examples: \( M = 6M_\odot \quad r_\bullet = 36 \text{ km} \)
\( 3M_\odot \quad 18 \text{ km} \)
\( 1M_\odot \quad 6 \text{ km} \) (except that a 1 solar mass star will end as a white dwarf—it's not massive enough to become a black hole!)

Summary: \( m_\star < 1.5 \text{ M}_\odot \) ends as a white dwarf
\( 1.5 \text{ M}_\odot < m_\star < 2.5 \text{ M}_\odot \) ends as a neutron star
\( m_\star > 2.5 \text{ M}_\odot \) ends as a black hole

All the above is for a single star—all bets are off if the star has a companion or multiple stars nearby. The presence of a companion star profoundly alters the evolution of the other star and changes the scenario.

Note that the above specified stellar final mass. It turns out stars are very clever about losing mass and thus alter their final stage. (For example, if an eight solar mass star loses 6 solar masses, it would avoid becoming a black hole, but instead would end up as a neutron star.) Ways a star can lose \( M_\odot \):

1) a huge solar wind, in which appreciable mass streams away from the star during its lifetime—more important for the very massive stars

2) planetary nebula—a star puffs off a shell of gas from the outer layers, leaving a hot central star. These appear as a small round disk (example the Ring Nebula in Lyra) or bubble-like object (example the Dumbbell Nebula in Vulpecula). These are nice telescopic objects. They are all about 40,000 years old when seen—before this they are too small, and after this they have expanded too thin to see

3) Nova—a sudden blast of radiant energy (or gain of 100 to 1000 times in luminosity) in a very short time. These usually occur in binary systems and are due to one of the two stars evolving into the giant phase and dumping mass onto its companion causing a mini-explosion.

4) Supernova—the mass of the star violently collapses and on rebound much of the mass is blasted into space in an explosion of radiant energy. Luminosity increases from
10⁶ to 10⁸ times. Most famous example is Crab Nebula. In most cases the core will then collapse to form a white dwarf or neutron star.

The last three objects listed above are examples of aperiodic variables, or sudden changes in the luminosity of a star (although it is believed planetary nebula and novae can re-occur for the same star, being rather benign). A more gentle kind of variable are the periodic variables of which there are many varieties, usually classified by details in their light curves. Variable stars represent an important but short-lived stage in the life cycle of most stars, between giant and terminal phases. (We are restricting ourselves to intrinsic variables here, with the light pulsating brighter and fainter due to internal changes in the star, as opposed to extrinsic variables which vary due to some external cause such as being eclipsed by a companion.)

Astronomers use the Julian Day Calendar to record the light curves of variable stars, where the Julian Day is the number of days elapsed since Greenwich mean noon of an arbitrary zero day beginning January 1, 4713 BC. For example January 1, 2000 is Julian Day 2,451,544.5 and you simply add one day per elapsed day after that. This avoids the complications of the calendar, making it easy to determine the period of a variable star by simply subtracting the relevant Julian Days between maxima.

We have time to mention only 2 of the many varieties of variables. RR Lyrae stars are a class of variable named for the prototype star in Lyra, and characterized by a unique light curve with period of a half day and ranges of about 1.5 magnitudes. They are mostly blue giants. The great thing about RR Lyrae stars is that they all have absolute magnitudes of about zero, thus you can find their distances using $M = m + 5 - 5 \log d (\text{pc})$. Hence stars are a very useful yardstick for measuring distances to external galaxies, provided you can find an RR Lyrae star there.

The second group of variables are the Cepheid Variables, named for their prototype δ Cepheid. These have a range of periods (1 to 50 days) and typically vary less than 1 magnitude. In 1910 Henrietta Leavitt found a useful relation called the Period-Luminosity Relation for Cepheids in the Magellanic Clouds (two nearby irregular galaxies seen in the southern hemisphere). So all you have to do is find a Cepheid variable in whatever distant galaxy you're studying, measure its period and read off its absolute magnitude from the graph—a very useful distance-determining technique.
We have been working our way out into the universe using various
distance-determining techniques, from simple trigonometric paral-
lex, to more indirect dynamic parallax and spectroscopic parallax
methods, and now use of the Period-Luminosity Relation.
We'll finish with one last method, used for the greatest
distances determined in the universe: Hubble's Law. In the 1920's
Hubble found that the light from distant galaxies is red-shifted
in proportion to their distance from us, the spectra from the fur-
therest objects being almost unrecognizable. Making the leap to
associate this redshift with a recessional velocity gives us the
picture of an expanding universe (remember the Doppler effect?).
We should quickly reassure ourselves this does not place us back
in the center of the universe, just because we see all the
galaxies receding from us--space itself is expanding, and all the
galaxies are receding from each other, the further apart they are
the higher the velocity of recession. So we have Hubble's Law:
\[ v = Hd, \]
where \( H \) is called Hubble's constant.
Hence for the most distant objects in the universe, we measure
their spectra, convert the redshifts to velocities and use
this relation to find their distances. There are problems
with this method, however: the Hubble constant is not that well
agreed upon, and some astronomers still argue that the quasar
redshifts are not caused by recession at great distances.
A big controversy today is whether the generally observed
expansion we see now will continue forever, or will the universe
slow down and then slowly recontract back to a giant fireball
again, similar to the original Big Bang from which it emerged
more than 10 billion years ago. But that's another lecture for
another day!