Announcements

Another fundamental question we ask about stars is about their brightness. In 120 BC Hipparchus in Greece made the first catalogue of 1048 naked eye stars, later edited by Ptolemy and tilted ‘Almagest’. As we noted earlier stars were first divided into 6 classes or magnitudes, and this was extended to include decimal magnitudes and also negative magnitudes for the brightest objects (e.g. sun’s m= -26.7). In 1820 John Herschel concluded the geometric progression in apparent brightness of stars is associated with an arithmetic progression of their magnitudes, in an attempt to quantify Hipparchus’ categories. This is now known as Fechner’s Law, that the (linear) eye response $R$ is proportional to the log of the stimulus $S$, a very general psychophysical phenomenon (including not only eye response, but the ear, sense of touch, etc). $R = c \log S$. The law does break down for very high or low intensities. Another way to state it is that equal increments in sensation are produced by equal ratios of stimulus.

In 1856 Pogson noted that a first magnitude star is exactly 100 times brighter than a sixth, which then lets us write for two stars of magnitudes $m$ and $n$, with luminosities $l_m$ and $l_n$, $m - n = 2.5 \log \left( \frac{l_n}{l_m} \right)$ where the luminosity is the rate of radiation into space by the star. (Remember a log is the power you must raise 10 to, to get the number in question, e.g. $\log 100 = 2$, $\log 1000 = 3$, etc.) Now we can generalize the small table we used earlier in the semester:

<table>
<thead>
<tr>
<th>$\Delta m$</th>
<th>magn. Difference</th>
<th>brightness ration ($l_n/l_m$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>~ 2.5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>~ 6</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>~ 16</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>~ 40</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>=100</td>
<td></td>
</tr>
</tbody>
</table>
To include any magnitude difference found (not just 1 to 5), simply break up the magnitude difference number into smaller numbers in the above table, and multiply each ratio:

For example, consider two stars differing in magnitude by 13 magnitudes (e.g. $m_1 = 20$ and $m_2 = 7$)

magnitude diff. $\Delta m = 13 = 5 + 5 + 3$
so brightness ratio $= (100) \times (100) \times 16 = 160,000$ and thus the $m = 7$ star is 160,000 times brighter than the $m = 20$ star.

Now there are problems with visual magnitudes, which are based on what the eye sees. The eye is not to reliable, and of course must be thoroughly dark-adapted. Then there’s the Perkinje effect: the sensitivity of the eye depends on color. In bright light red appears brighter than blue, but in dim light (when you use the rods of the eye which are sensitive only to blue light), then blue appears to be brighter than red. For example among the brighter stars, Rigel is blue and Betelgeuse is red—which is brighter? Thirdly, the eye cannot integrate add up light – it either sees a dim object or not; it is an instantaneous detector. Eye estimates of brightness are called visual magnitudes, $m_v$.

It is better to use photographs which can have long exposures and thus integrate up the light; these are typically quoted for one specific wavelength. So we have photographic magnitudes, $m_p$, picked to measure e.g. the blue light of stars (whereas the eye measures magnitudes in the yellow-green range where the eye is most sensitive). A star’s brightness is usually measured by the size of the image on the photographic plate when plates are used. (There are also many new devices, such as light meters on your camera or newer CCD detectors for determining magnitudes.)

Astronomers define color index $CI = m_p - m_v$, the difference between photographic and visual magnitudes. This will turn out to be a very important parameter; for example it is independent of the distance to the star!

All of the above is for relative magnitudes as seen in the night sky. How can we really compare stars when they lie at different distances away; for example, a very bright star like Rigel is not the brightest star in our sky because it is much further away than the sun, our brightest ‘star’. To correctly compare stars, they must all lie at the same distance, so we must calculate what their brightness would then be at some standard distance. Astronomers arbitrarily picked that standard distance to be 10 parsecs (where a star would have parallax = 0.1”).

Definition: the Absolute Magnitude of a star is the apparent magnitude it would have if located at a distance of 10 pc.

Now there is a relation between absolute M (magnitude if it were 10pc away) and apparent magnitude $m$ (as seen by us now) : $M = m + 5 - 5 \log R$, where $R$ is distance in parsecs

E.g. for Sirius $m = -1.52$, $R = 2.66$pc, $\log R = 0.42$ so $M = -1.52 + 5 - 5 \times 0.42 = 1.36$
Where did this equation come from? Basically there are two ingredients, the inverse square law for the diminution of light due to distance, and secondly, the basic magnitude scale. Maybe it’s clearer if you write \( M - m = -2.5 \log(r^2/10^2) \) and then expand out this expression (and remember luminosity is proportional to \( 1/r^2 \)). Let’s check the equation for a specific case, e.g. for \( R = 1 \) pc. \( M = m + 5 - 5\log 10 = m \) as you expect from the definition of absolute magnitude.

If a star is 10 pc away, its \( M = m \).
If a star is nearer than 10 pc away, \( m < M \) (it appears brighter, so \( m \) is a smaller number than \( M \), remembering the ‘backwards’ brightness scale)
If a star is further than 10 pc away, \( m > M \) (it appears dimmer than if it were at 10 pc so \( m \) is a bigger number than \( M \)).

What is the absolute magnitude of the sun? Ans. 4.8, hardly visible in our sky. (To get this you must use the sun’s \( m = -26.7 \) and express its distance \( R = 1 \) AU in parsecs.) Now we can truly compare the sun with Sirius, whose \( M = 1.3 \) above. The magnitude difference is 4.8 - 1.3 = 3.5. A magnitude difference of 3 is a brightness ratio of 16, and for a difference of 4 the ratio is 40, so in fact for a difference of 3.5 the answer is 30: Sirius has 30 times the light output of the sun.